Cap ① ⟶ REVISÃO DE VETORES

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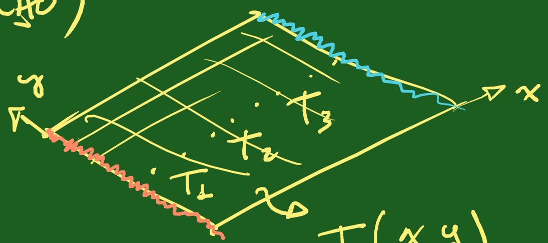
↳ Grandezas escalares: massa (kg), densidade (kg/m^3), temperatura (°C, K), potencial (V/m), etc. (1 informação)

1 kg, 23 °C,



$T(x,y)$

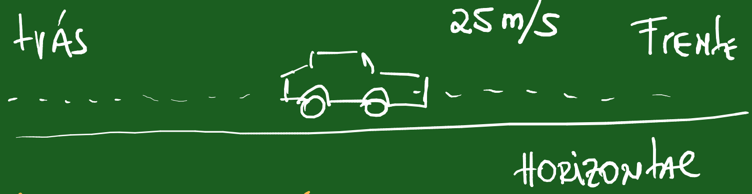
↳ Grandezas vetoriais (3 informações)

① Intensidade/módulo
② Direção (linha de ação)
③ Sentido (para onde aponta)

↳ obrigatório

$|\vec{v}| = \boxed{25 \frac{m}{s}}$ $\frac{[L]}{[T]}$, $|\vec{F}| = 20\,N$, $|\vec{a}| = 2\frac{m}{s^2}$

$|\vec{E}| = 10\frac{V}{m}$, $|\vec{B}| = 10\,T$



② Representação

$\boxed{\vec{v}}$, $\boxed{\vec{A}}$, $\boxed{\vec{B}}$, $\boxed{\vec{C}}$,

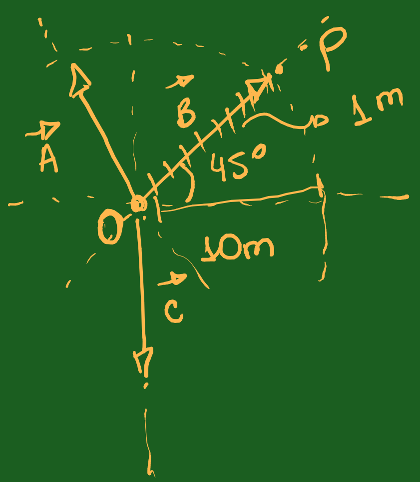
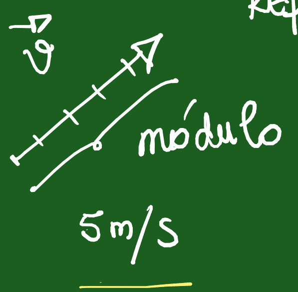
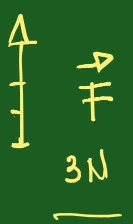


Diagrama de vetores

Representação visual



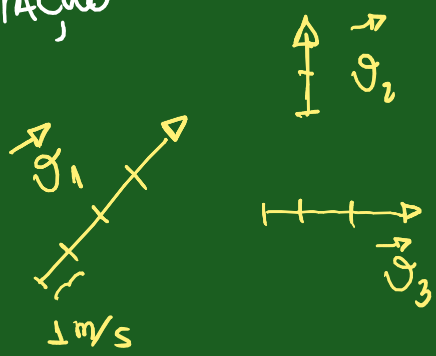


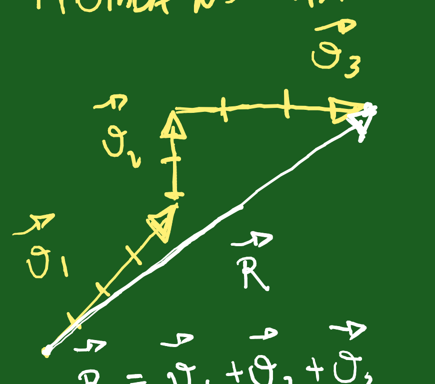
* Vetores de dif. unidades não podem sofrer adição

③ Operações com vetores

→ Adição/subtração

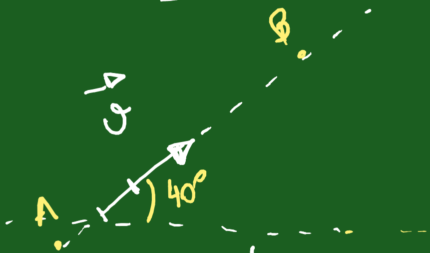
tromba no rabinho





$\vec{R} = \vec{v}_1 + \vec{v}_2 + \vec{v}_3$

operação ↦ Escalar x Vetor



$\to |\vec{v}| = 2\,m.s^{-1} = v$

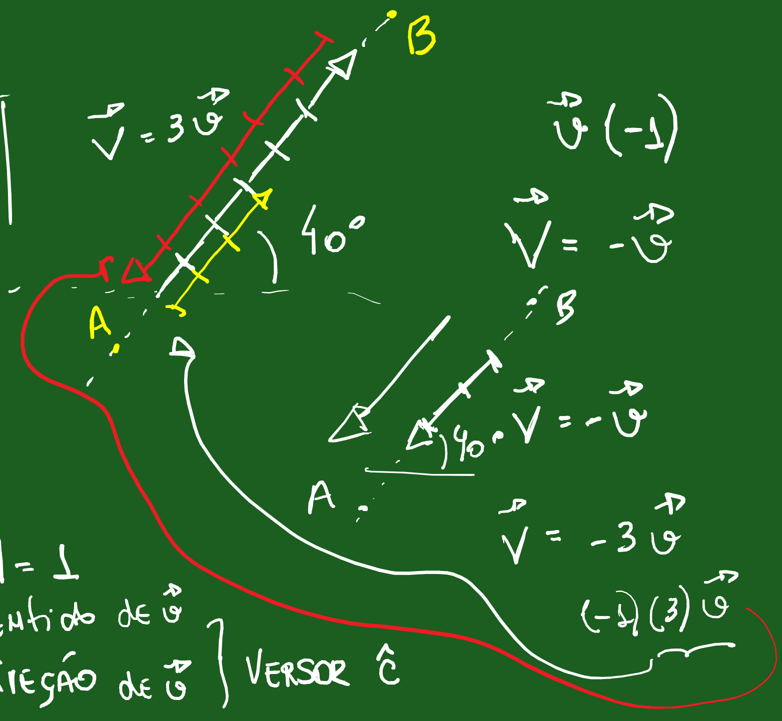
→ direção (40° c/ horizontal)

→ sentido do ponto A ao ponto B

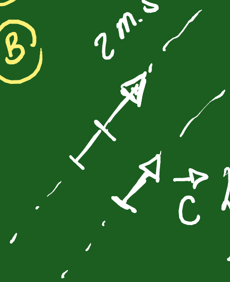
$$\boxed{3\vec{v} = \vec{V}}$$

$$\boxed{\vec{V} = 3\vec{v}}$$

$|\vec{V}| = 3|\vec{v}| = 3 \times 2 = 6\,m.s^{-1}$



$\vec{c} = \dfrac{1}{|\vec{v}|}\vec{v} = \dfrac{\vec{v}}{|\vec{v}|}$



$\vec{c}$ { $|\vec{c}| = 1$, sentido de $\vec{v}$, direção de $\vec{v}$ } Versor $\hat{c}$

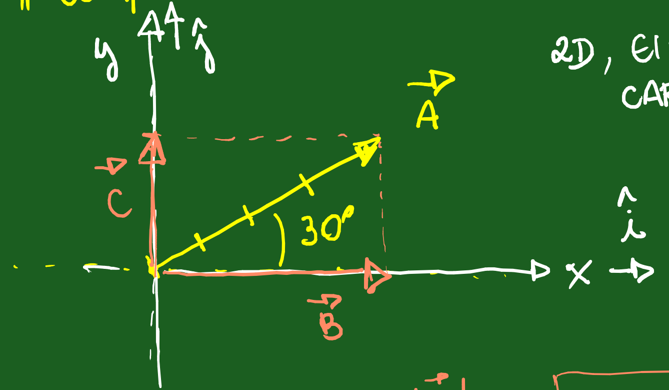
$\hat{c}$ ↦ Versor, vetor unitário (módulo 1 s/ unidade)
↳ direção
sentido

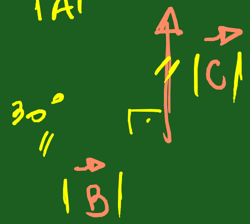
$$\boxed{\vec{v} = 2(m.s^{-1})\,\hat{c}}$$

→ representação vetorial na forma analítica/algébrica

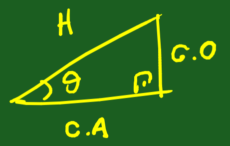
Componentes de um vetor

2D, eixos cartesianos





$|\vec{A}|^2 = |\vec{C}|^2 + |\vec{B}|^2$



$\operatorname{sen}\theta = \dfrac{C.O}{H}$, $\cos\theta = \dfrac{C.A}{H}$

$\operatorname{tg}\theta = \dfrac{C.O}{C.A}$

ortogonal

$\operatorname{Sen}30° = \dfrac{|\vec{C}|}{|\vec{A}|}$, $\quad \boxed{|\vec{C}| = |\vec{A}|.\operatorname{sen}30°}$

$\operatorname{Cos}30° = \dfrac{|\vec{B}|}{|\vec{A}|}$, $\quad \boxed{|\vec{B}| = |\vec{A}|.\operatorname{Cos}30°}$

$$\boxed{\vec{A} = \vec{B} + \vec{C}}$$

$\vec{B}$ ≡ componente do vetor $\vec{A}$ na dir. (x) $= \vec{A}_x$

$\vec{C}$ ≡ componente do " $\vec{A}$ " " (y) $= \vec{A}_y$

$\vec{B} = |\vec{B}|\,\hat{i}$

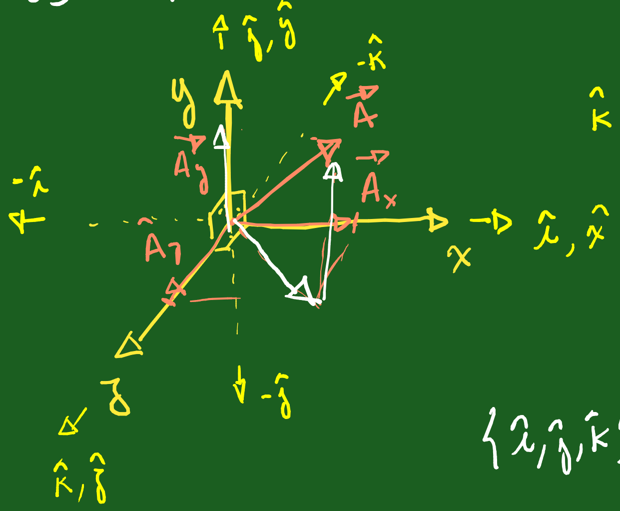
$\vec{C} = |\vec{C}|\,\hat{j}$

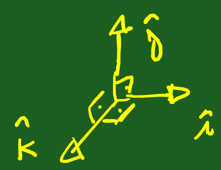
$\vec{A} = |\vec{B}|\hat{i} + |\vec{C}|\hat{j} = \vec{A}_x + \vec{A}_y = A_x\hat{i} + A_y\hat{j} =$

$\vec{A} = A\cos 30°\,\hat{i} + A\operatorname{Sen}30°\,\hat{j}$, $\quad \hat{i} = \hat{x}$, $\hat{j} = \hat{y}$, $\hat{k} = \hat{z}$

Representação de Componentes

3D ⟼ "plano cartesiano"





BASE ORTONORMAL

ORTOGONAL + NORMALIZADA

$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$

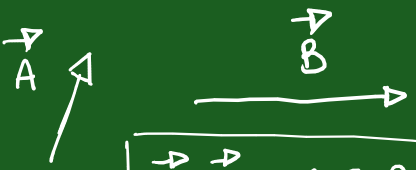
$\vec{A}_x + \vec{A}_y + \vec{A}_z = \vec{A}$ $\{\hat{i},\hat{j},\hat{k}\}$

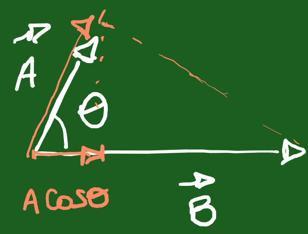
$$\boxed{\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}$$

$\{\hat{i},\hat{j},\hat{k}\}$ ⟼ tríade (regra da mão direita)

OPERAÇÕES c/ VETORES

Produto escalar ⟼





$$\boxed{\vec{A}\cdot\vec{B} = A.B.\cos\theta}$$

$\vec{A}\cdot\vec{B} = A\cos\theta . B$

$\vec{A}\cdot\vec{B} = B\cos\theta . A$ (escalar)

$0 < \theta < 360$

$0 < \theta < 2\pi$

$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A} = AB\cos\theta$

$\vec{A} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$\vec{B}\cdot\vec{A} = (A_x\hat{i})\cdot(B_x\hat{i}) + A_x\hat{i}\cdot B_y\hat{j} + A_x\hat{i}\cdot B_z\hat{k} + A_y\hat{j}\cdot B_x\hat{i} + A_y\hat{j}\cdot B_y\hat{j} + \dots$

$\alpha\vec{A}\cdot\vec{B} = \vec{A}\cdot(\alpha\vec{B}) = \vec{A}\cdot\vec{B}\,\alpha$

$A_xB_x\underbrace{\hat{i}\cdot\hat{i}}_{1} + A_xB_y\underbrace{\hat{i}\cdot\hat{j}}_{0} \dots$

$\hat{i}\cdot\hat{i} = |\vec{i}||\vec{i}|.\cos 0 = 1$

$\hat{i}\cdot\hat{j} = 1.1.\cos 90° = 0$

$\hat{i}\cdot\hat{k} = 0$

$\hat{i}\cdot\hat{i} = 1 = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

$\hat{i}\cdot\hat{j} = \hat{i}\cdot\hat{k} = \hat{j}\cdot\hat{k} = 0$ (ortonormal)

$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$ = escalar.

exemplo:

$\vec{F}\cdot\vec{d} = W$ (trabalho)

$\vec{A}\cdot\hat{i} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot\hat{i} = A_x\hat{i}\cdot\hat{i} = A_x$, $A_y = \vec{A}\cdot\hat{j}$, $A_z = \vec{A}\cdot\hat{k}$

* Produto vetorial $\vec{A}\times\vec{B} = \vec{C}$, $\vec{A}\wedge\vec{B} = \vec{C}$ ⟼ $\vec{C} = \vec{A}\times\vec{B}$; $\vec{B}\times\vec{A} = -\vec{C}$

$\vec{A}\times\vec{B} = \vec{C}$ | $|\vec{C}| = |\vec{A}||\vec{B}|\,\mathrm{sen}\theta$

$|\hat{i}\times\hat{i}| = |\hat{i}||\hat{i}|.\cancel{\mathrm{sen}\theta}^{0}$

$\hat{i}\times\hat{i} = 0$

$\hat{j}\times\hat{j} = 0$

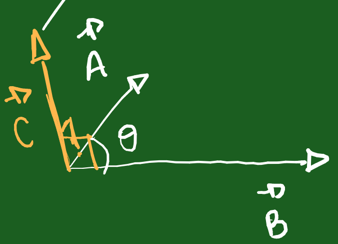
$\hat{k}\times\hat{k} = 0$

$$\boxed{\hat{i}\times\hat{j} = \hat{k}}$$

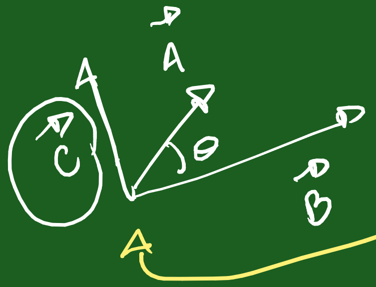
$\vec{A}\times\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\times(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$A_x\hat{i}\times B_y\hat{i} + A_x\hat{i}\times B_y\hat{j} + \dots$

$A_xB_y(\underbrace{\hat{i}\times\hat{i}}_{0}) + A_xB_y\,\hat{i}\times\hat{j}$



$$\vec A \times \vec B = \begin{vmatrix} \hat i & \hat j & \hat k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \vec C$$



$$\hat i (A_y B_z - A_z B_y) + (-)\hat j (A_x B_z - A_z B_x) + \hat k \quad B_y - A_y B_x)$$

$$\vec A \times \vec B = \underbrace{(A_y B_z - A_z B_y)\hat i}_{C_x \hat i} + \underbrace{(A_z B_x - A_x B_z)\hat j}_{C_y \hat j} + \underbrace{(A_x B_y - A_y B_x)\hat k}_{C_z \hat k} = \vec C$$

$$C_x \hat i + C_y \hat j + C_z \hat k = \vec C$$

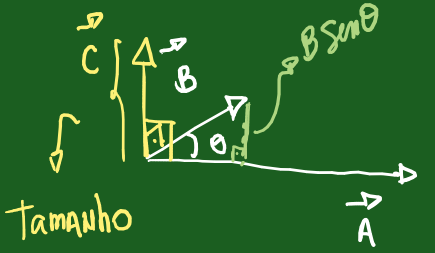
Produto Vetorial

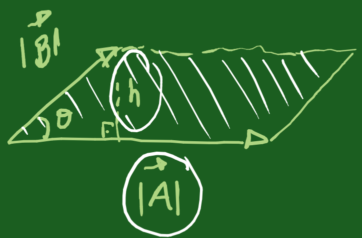
$\vec C = \vec A \wedge \vec B$ ou $\vec C = \vec A \times \vec B$, $\quad \vec B \times \vec A = -\vec C$

$$|\vec C| = |\vec A \times \vec B| = AB\,\text{Sen}\,\theta = \underbrace{|\vec A||\vec B|\,\text{Sen}\,\theta}_{} = A\,\underbrace{B\,\text{Sen}\,\theta}_{h}$$

$$\vec C \cdot \vec A = 0 = \vec C \cdot \vec B = CB\cos\theta$$

$|\vec C|$ = "ÁREA" do paralelogramo

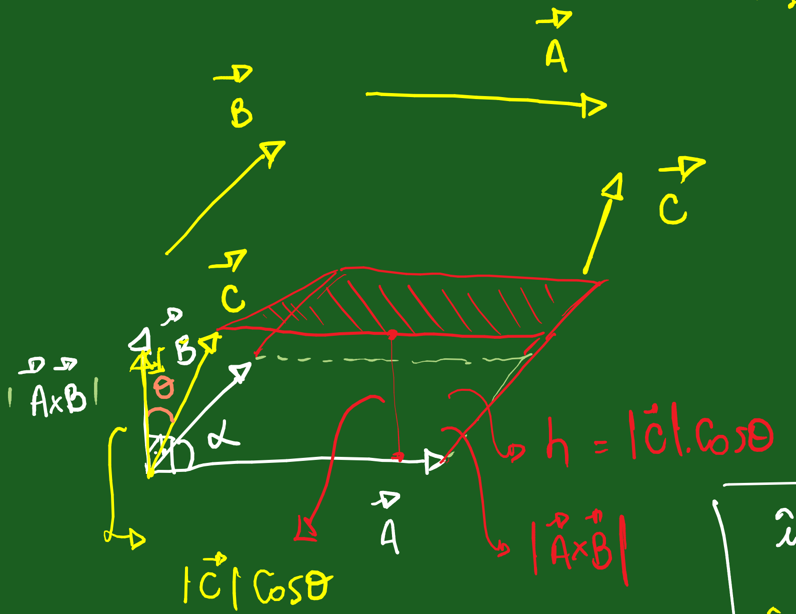




Produto misto

$\vec A, \vec B$ e $\vec C$

$$(\vec A \times \vec B) \cdot \vec C = |\vec A \times \vec B||\vec C|\cos\theta$$
$$= AB\,\text{Sen}\,\alpha . C . \cos\theta$$
$$= ABC.\,\text{Sen}\,\alpha \cos\theta$$
$$= |\vec A \times \vec B||\vec C| . \cos\theta$$

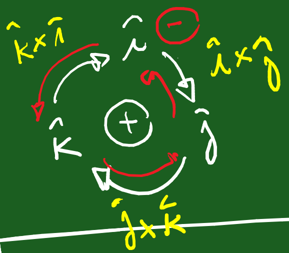


$h = |\vec C| . \cos\theta$

$|\vec A \times \vec B||\vec C| . \cos\theta$ = "Volume" //

$\hat i \times \hat j = \hat k$, $\hat j \times \hat k = \hat i$, $\hat k \times \hat i = \hat j$

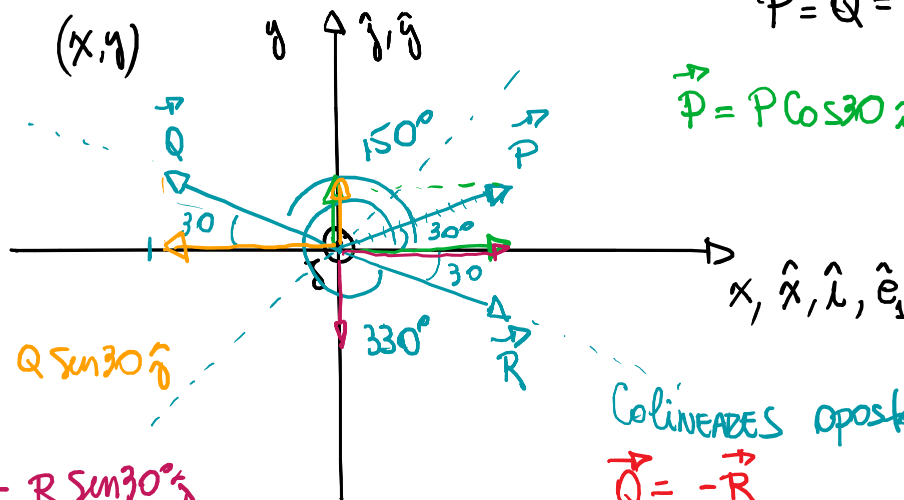
$\hat j \times \hat i = -\hat k$



Três vetores, $\vec{P}$, $\vec{Q}$ e $\vec{R}$, com módulos iguais a 100 m, estão no plano xOy e formam, respectivamente, ângulos de 30°, 150° e 330° com o sentido positivo do eixo x, sentido anti-horário. Determine módulo, direção e sentido do vetor que é resultado da soma desses três vetores.

$P = Q = R = 100m$

PLANO CARTESIANO (x,y)



$\vec{P} = P\cos 30\,\hat{i} + P\,\text{Sen}\,30\,\hat{j}$

$\vec{Q} = -Q\cos 30\,\hat{i} + Q\,\text{Sen}\,30\,\hat{j}$

$\vec{R} = R\cos 30\,\hat{i} - R\,\text{Sen}\,30°\,\hat{j}$

Colineares opostos

$\vec{Q} = -\vec{R}$

$\vec{Q} + \vec{R} + \vec{P} = -\vec{R} + \vec{R} + \vec{P} = \vec{P}$

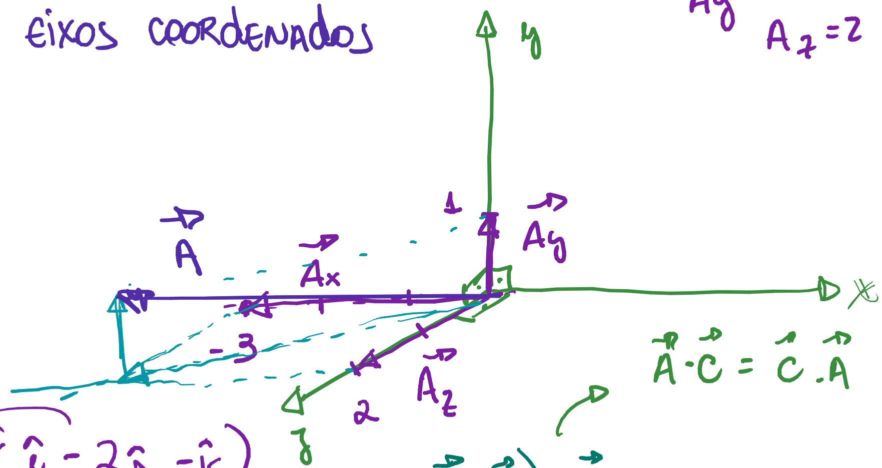
$$\vec{P}+\vec{Q}+\vec{R} = P\cos 30°\,\hat{i} + \cancel{P\,\text{Sen}\,30°}\,\hat{j} - \cancel{Q\cos 30°}\,\hat{i} + Q\,\text{Sen}\,30°\,\hat{j} + R\,\cancel{\cos 30}\,\hat{i} - \cancel{R\,\text{Sen}\,30}\,\hat{j}$$

$$= P\cos 30°\,\hat{i} + \underset{\overset{\parallel}{P}}{Q}\,\text{Sen}\,30°\,\hat{j} = P\cos 30°\,\hat{i} + P\,\text{Sen}\,30°\,\hat{j} = \vec{P}$$

7) Dados os vetores $\vec{A} = -3\vec{i} + \vec{j} + 2\vec{k}$; $\vec{B} = \vec{i} + \vec{j} + \vec{k}$; $\vec{C} = \vec{i} - 2\vec{j} - \vec{k}$; determine:

$A_x = -3$
$A_y = 1$
$A_z = 2$

eixos coordenados



a) $\vec{A} - 2\vec{B} + 3\vec{C}$;

b) $(\vec{A} + \vec{B})\cdot\vec{C}$;

c) $\vec{A}\times(\vec{B} + \vec{C})$;

$\vec{A} - 2\vec{B} + 3\vec{C}$

$-3\hat{i} + 1\hat{j} + 2\hat{k} - 2(\hat{i} + \hat{j} + \hat{k}) + 3(\hat{i} - 2\hat{j} - \hat{k})$

$-3\hat{i} - 2\hat{i} + 3\hat{i} + 1\hat{j} - 2\hat{j} - 6\hat{j} + 2\hat{k} - 2\hat{k} - 3\hat{k}$

$\boxed{-2\hat{i} - 7\hat{j} - 3\hat{k} = \vec{A} - 2\vec{B} + 3\vec{C}}$

$\vec{A}\cdot\vec{C} = \vec{C}\cdot\vec{A}$

b) $(\vec{A} + \vec{B})\cdot\vec{C}$ = Escalar

$\vec{A}\cdot\vec{C} + \vec{B}\cdot\vec{C}$ = Escalar

$(-3\hat{i} + 1\hat{j} + 2\hat{k})\cdot(\hat{i} - 2\hat{j} - \hat{k})$

$-3 - 2 - 2$

-7

c) $\vec{A}\times(\vec{B} + \vec{C}) = \vec{A}\times\vec{B} + \vec{A}\times\vec{C}$ = vetor

$\cancel{\vec{B}\times\vec{A}}$

$\vec{A}\times\vec{C} = \left|\begin{array}{ccc|cc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ A_x & A_y & A_z & A_x & A_y \\ C_x & C_y & C_z & C_x & C_y \end{array}\right|$

8) Na Figura 1.16, $\vec{A}$ e $\vec{B}$ estão contidos no plano xOy sendo que $A = 10$ un e $B = 12$ un. Determine:

a) $\vec{A}\cdot\vec{B}$;

b) $\vec{A}\times\vec{B}$.

$\vec{A} = A\cos 60\hat{i} + A\sin 60°\hat{j}$

$\vec{B} = B\hat{i}$

$\vec{A}\cdot\vec{B} = A.B.\cos\theta$

$= 10\times 12.\cos 60°$

$= 10.12.\frac{1}{2} = 10\times 6 = 60$

$\boxed{\vec{A}\cdot\vec{B} = 60\ \text{un}^2}$

$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 5\sqrt{3} & 0 \\ 12 & 0 & 0 \end{vmatrix}$

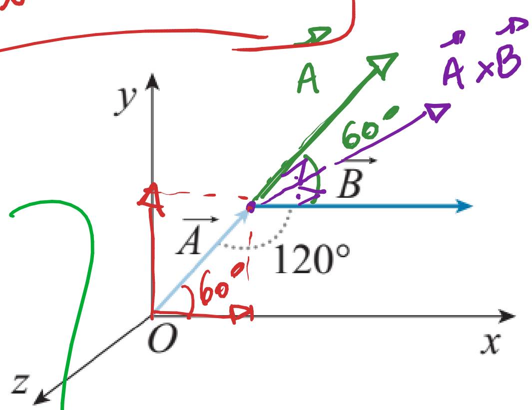


Figura 1.16

$\hat{i}(\cancel{0}) - \hat{j}(\cancel{0}) + \hat{k}(-12\times 5\sqrt{3})$

$-60\sqrt{3}\hat{k}$

$\boxed{\vec{A}\times\vec{B} = -60\sqrt{3}\hat{k}}$

$\vec{A}\times\vec{B} \equiv$ vetor!!

$|\vec{A}\times\vec{B}| = A.B.\sin\theta = 10.12.\sin 60 = 10\times 12\frac{\sqrt{3}}{2} = 60\sqrt{3}$